The NKM with a Supply Shock: Matrix Representation

Macroeconomics (M8674), May 2026

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The New Keynesian Model with a supply shock

The linearized baseline version of the NKM with supply shocks can be written with 7 equations:

$$
\begin{aligned}
&\text{IS}: && \hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \tfrac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r^n\right) \\
&\text{Taylor rule}: && i_t = \pi_t + r^n + \phi_\pi\left(\pi_t - \pi^T\right) + \phi_y \cdot \hat{y}_t \\
&\text{AS}: && \pi_t = \kappa \cdot \hat{y}_t + \beta \cdot \mathbb{E}_t \pi_{t+1} + s_t \\
&\text{Supply shock}: && s_t = \rho_s s_{t-1} + \varepsilon_t^s \quad , \quad \varepsilon_t^s \sim \mathcal{N}(0,1) \\
&\text{Output allocation}: && \hat{y}_t = \hat{c}_t \\
&\text{Labor supply}: && \hat{\ell}_t = \left(\tfrac{1-\sigma}{1+\gamma}\right)\hat{y}_t \\
&\text{Technology}: && \hat{a}_t = \left[1 - \tfrac{\alpha(1-\sigma)}{1+\gamma}\right]\hat{y}_t
\end{aligned}
$$

Variables and parameters

- **Endogenous variables**: $\{\hat{y}, \pi, i, \hat{c}, s, \hat{\ell}, \hat{a}\}$ are, respectively, the output-gap, inflation, nominal interest rate, the consumption-gap, supply shock, labor supply, and technology.
- **Exogenous variables**: $\{r^n, \pi^T, \varepsilon\}$, which represent, respectively, the natural level of the real interest rate, the target inflation rate, and a random disturbance.
- **Parameters**: $\{\sigma \ , \ \phi_\pi \ , \ \phi_y \ , \ \kappa = \frac{\psi(1-\mu)(1-\mu\beta)}{\mu} \ , \ \psi \ , \ \beta \ , \ \mu \ , \ \rho_s \ , \ \alpha \ , \ \gamma\}$
- **Forward-looking variables**: $\hat{y}_t \ , \ \pi_t$
- **Backward-looking variables**: s_t
- **Static variables**: $i_t \ , \ \hat{\ell}_t \ , \ \hat{a}_t \ , \ \hat{c}_t$

4 equations vs 4 unknowns

- The model can be fully simulated only with 4 equations and 4 unknowns:

$$
\begin{aligned}
\text{Supply shock :} \quad & s_{t+1} = \rho_s s_t + \varepsilon^s_{t+1} \\
\text{Taylor rule :} \quad & i_{t+1} = \pi_{t+1} + r^n + \phi_\pi(\pi_{t+1} - \pi^T) + \phi_y \cdot \hat{y}_{t+1} \\
\text{AS :} \quad & \pi_t = \kappa \cdot \hat{y}_t + \beta \cdot \mathbb{E}_t \pi_{t+1} + s_t \\
\text{IS :} \quad & \hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1} - r^n)
\end{aligned}
$$

- The first two are non-forward looking variables, and the last two are forward looking variables.
- To separate the two blocks of equations, the order we write them matters:
  - First write down the non-forward looking block
  - Then write down the forward looking block

Matrix representation

- The 4 equations can be written as:

$$1s_{t+1} + 0i_{t+1} + 0\mathbb{E}_t\pi_{t+1} + 0\mathbb{E}_t\hat{y}_{t+1} = \rho_s s_t + 0i_t + 0\pi_t + 0\hat{y}_t \quad + 1\varepsilon^s_{t+1}$$

$$0s_{t+1} + 1i_{t+1} - (1+\phi_\pi)\mathbb{E}_t\pi_{t+1} - \phi_y\mathbb{E}_t\hat{y}_{t+1} = 0s_t + 0i_t + 0\pi_t + 0\hat{y}_t \quad + 0\varepsilon^i_{t+1} + r^n - \phi_\pi\pi^T$$

$$0s_{t+1} + 0i_{t+1} + \beta\mathbb{E}_t\pi_{t+1} + 0\mathbb{E}_t\hat{y}_{t+1} = -1s_t + 0i_t + 1\pi_t - \kappa\hat{y}_t \quad + 0\varepsilon^\pi_{t+1}$$

$$0s_{t+1} + 0i_{t+1} + (1/\sigma)\mathbb{E}_t\pi_{t+1} + 1\mathbb{E}_t\hat{y}_{t+1} = 0s_t + (1/\sigma)i_t + 0\pi_t + 1\hat{y}_t + 0\varepsilon^y_{t+1} + (1/\sigma)r^n$$

- Passing the equations into matrices gives:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -(1+\phi_\pi) & -\phi_y \\ 0 & 0 & \beta & 0 \\ 0 & 0 & \frac{1}{\sigma} & 1 \end{bmatrix}}_{\mathcal{A}} \begin{bmatrix} s_{t+1} \\ i_{t+1} \\ \mathbb{E}_t\pi_{t+1} \\ \mathbb{E}_t\hat{y}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \rho_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & -\kappa \\ 0 & \frac{1}{\sigma} & 0 & 1 \end{bmatrix}}_{\mathcal{B}} \begin{bmatrix} s_t \\ i_t \\ \pi_t \\ \hat{y}_t \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\mathcal{C}} \begin{bmatrix} \varepsilon^s_{t+1} \\ \varepsilon^i_{t+1} \\ \varepsilon^\pi_{t+1} \\ \varepsilon^y_{t+1} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ r^n - \phi_\pi\pi^T \\ 0 \\ (1/\sigma)r^n \end{bmatrix}}_{\mathcal{D}}$$

The model is ready for the computer